Annexure-1 : Generation of time histories and Power spectral density function (PSDF) compatible with Design Response Spectrum (DRS)

A. 1. 0 Generation of Power spectral density function (PSDF) compatible with Design Response Spectrum (DRS)

Spectrum compatible time history is generated from Power spectral density function (PSDF) which is compatible with design response spectrum. PSDF can be generated directly from response spectrum using two methods. The first method is termed as method-SIMQK which is used in the SIMQK program for artificial motion generation. The details of this method to obtain PSDF $\phi(\omega_0)$ from smooth target velocity response spectrum (R_v) are given below:

PSDF $\phi(\omega_0)$ is given by,

$$\phi(\omega_0) = \frac{1}{\omega_0\left(\dfrac{\pi}{4\beta_T} - 1\right)}\left[\frac{\omega_0^2 R_v^2}{F_0^2} - \int_0^{\omega_0} \phi(\omega)d\omega\right]^{1/2}$$

Where F_0 is the factor of multiplication with standard deviation to obtain the peak response of SDOF and it is given as

$$F_0(\omega_0) = \left[2\log\left\{2n\left[1 - \exp\left(-\delta_y(T)\sqrt{\pi\log 2n}\right)\right]\right\}\right]^{\frac{1}{2}}$$

where,

$$n = \left(\Omega_y(T)T_0 / 2\pi\right)\left(-\log p\right)^{-1}$$

Ω_y and δ_y are the response spectral moments and given by:

$\Omega_y(T) = \Omega$ and $\delta_y(T) = \delta_y$ at very low periods (response spectral moments are equal to those of the ground motion)

$$\Omega_y(T) = \omega_0 \text{ and } \delta_y = \left(\frac{4\beta_T}{\pi}\right)^{1/2}$$

p is the probability of exceedance
T is the earthquake time duration

$$\beta_T = \frac{\beta}{1 - e^{-2\beta\omega_0 T}}$$

The second method is termed as method-UNRUH_KANA which is described below:

Power Spectral Density Function (PSDF) $\psi(\omega)$ of Single degree of Freedom System subjected to excitation PSDF $\phi(\omega)$ is given as

$$\psi(\omega, \omega_0) = H(\omega, \omega_0) \times H^*(\omega, \omega_0) \times \phi(\omega)$$

Where $H(\omega, \omega_0) = \left[\dfrac{\omega_0^2 + 2i\omega_0\beta\ \omega}{\left(\omega_0^2 - \omega^2\right) + 2i\omega_0\beta\ \omega}\right]$

and $H^*(\omega, \omega_0)$ is the complex conjugate transfer function.

Now $\psi(\omega, \omega_0) = \left[\dfrac{\omega_0^4 + 4\omega_0^2\beta^2\omega^2}{\left(\omega_0^2 - \omega^2\right)^2 + 4\omega_0^2\beta^2\omega^2}\right]\phi(\omega)$

RMS value or standard deviation of the response of SDOF having frequency ω_0 is given as

$$\sigma^2(\omega_0) = \int_0^{\omega_{max}} \psi(\omega, \omega_0)d\omega$$

Response of SDOF in terms of standard deviation is given as

$$R(\omega_0) = F_0(\omega_0) \times \sigma(\omega_0)$$

Where F_0 is the factor of multiplication with standard deviation to obtain the peak response of SDOF and it is given as

$$F_0(\omega_0) = \left[-2\ln\left\{ -\left(\frac{\pi}{T}\right)\left(\frac{\sigma}{\dot{\sigma}}\right)\ln(1-p)\right\} \right]^{\frac{1}{2}}$$

Where T is the earthquake time duration and

$$\dot{\sigma}^2(\omega_0) = \int_0^{\omega_{max}} \omega^2 \psi(\omega,\omega_0)d\omega \quad \text{and}$$

p is the probability of exceedance. Typical value is 0.15

To have compatible mapping between and earthquake response acceleration and PSDF, an approximate solution is given as

$$\hat{\phi}(\omega_0) = \frac{2\beta}{\pi\omega_0} R^2(\omega_0) \times \left\{ -2\ln\left[-\left(\frac{\pi}{\omega_0 T}\right)\ln(1-p)\right] \right\}^{-1}$$

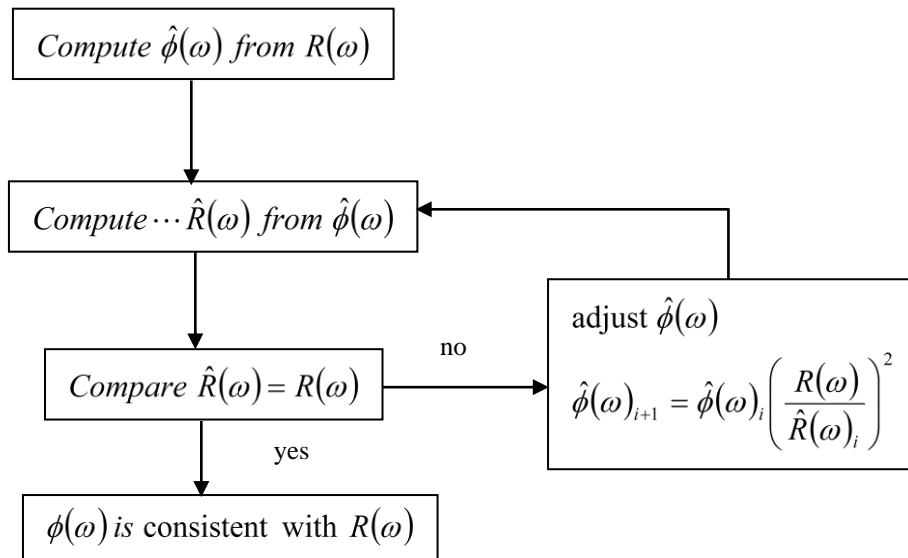is used as an initial estimate in an iterative procedure for which flow chart is given as follows.

Fig. A. 1 Flow chart for the generation of PSDF

A. 1. 1 Procedure for generation of spectrum compatible time histories

Periodic function can be expanded into a series of sinusoidal waves

$$X(t) = \sum_n A_n \sin(\omega_n t + \phi_n)$$

A_n is the amplitude and ϕ_n is the phase angle of the nth contributing sinusoid. Random number generator is used to produce strings of phase angles in the range between 0 and 2π. The amplitudes A_n are related to the (one sided) spectral density function Ø(ω) as:

$$\phi(\omega_n)\Delta\omega = \frac{A_n^2}{2}$$

To simulate the transient character of real earthquakes, the steady state motions are multiplied by deterministic envelop function I(t). The artificial motion becomes:

$$Z(t) = I(t)\sum_n A_n \sin(\omega_n t + \phi_n)$$

To smoothen the calculated spectrum and to improve the matching an iterative procedure is generally used. To do this the PSD is modified as follows:

$$\phi(\omega)_{i+1} = \phi(\omega)\left[\frac{S_{v(\omega)v}}{S_{v(\omega)}^i}\right]^2$$

where S_v is the target spectral value.

Annexure-2: Method of analysis

A.2.1 The Number of Modes Considered in the Modal Superposition Method

Number of modes included in the analysis shall be sufficient to ensure that inclusion of all remaining modes does not result in more than 10% increase in total responses of interest. Alternatively, ASCE standard (4-98) permits to include all the modes in the analysis having frequencies less than the ZPA frequency or cut-off frequency provided that the residual rigid response due to the missing mass is included.

The following criteria to be adapted while choosing the minimum number of modes to be considered.

1. Number of modes extracted are such that highest mode corresponding to a frequency greater than or equal to 33 Hz.

2. The numbers of modes extracted are such that the cumulative modal mass is more than 90% in each of the three directions.

Any one of the two methods can be used to determine the no of modes to be considered in modal superposition analysis.

A.2.2 Combination of Modal Response

  i. SRSS method
 ii. 10% method
iii. Double Sum method
 iv. CQC method

(i) With No Closely Spaced Modes (SRSS)

In a response spectrum modal dynamic analysis if the modes are not closely spaced (two consecutive modes are defined as closely spaced if their frequency differ from each other by 10% or less of the lower frequency) the representative maximum value of particular response of interest for design should be obtained by taking the square root of the sum of the squares (SRSS). Mathematically this can be expressed as follows.

$$R = \left[\sum_{k=1}^{N} R_k^2 \right]^{1/2}$$

Where R is the representative max value of particular response of a given element to a given component of an earthquake and Rk is the peak value of the response of the element due to the k th mode and N is the number of significant modes considered in the modal response combination.

(ii) With Closely Spaced Modes (10% methods)

In a response spectrum modal dynamic analysis if the modes are closely spaced (two consecutive modes are defined as closely spaced) if their frequency differ from each other by 10% or less of the lower frequency) the representative maximum value of particular response of interest should be obtained by taking the square root of the sum of the squares (SRSS). Mathematically this can be expressed as follows.

$$R = \left[\sum R_k^2 + 2\sum |R_i R_j| \right]^{1/2} \quad i \neq j$$

$$\frac{\omega_j - \omega_i}{\omega_i} \leq 0.1$$

$$also \, 1 \leq i \leq j \leq N$$

Where R is the representative max value of particular response of a given element to a given component of an earthquake and R_k is the peak value of the response of the element due to the kth mode and N is the number of significant modes considered in the modal response combination.

(iii) Double sum method

$$R = \left[\sum_{k=1}^{N} \sum_{s=1}^{N} |R_k\, R_s\,| \, \varepsilon_{ks} \right]^{1/2}$$

where,

$$\varepsilon_{ks} = \left[1 + \left[\frac{(\omega_k{}' - \omega_s{}')}{\xi_k{}'\omega_k + \xi_s{}'\omega_s} \right] \right]^{-1}$$

in which

$$\omega_k{}' = \omega_k \sqrt{(1 - \xi_k{}^2)}$$

$$\xi_k{}' = \xi_k + \frac{2}{t_d\, \omega_k}$$

where ω_k, ξ_k are the modal frequency and the damping ratio in the kth mode respectively, and t_d is the duration of the earthquake.

The above equation for $\varepsilon_{ks} = 1$ for k=s. Consider two modes with equal frequencies and damping values then $e_{ks} = 1$ and equation gives

$R^2 = R_1{}^2 + R_2{}^2 + 2R_1 R_2$

$R = R_1 + R_2$

The double sum rule correctly gives the combined response as the algebraic sum of the two modal response values. On the other hand, if the two modes had sufficiently separated frequencies $\varepsilon_{ks} = 0$ and we would get

$R = (R_1{}^2 + R_2{}^2)^{1/2}$

(iv) Complete Quadratic Method

This method does not take in to account the duration of the earthquake.

$$R = \left[\sum_{k=1}^{N} \sum_{s=1}^{N} R_k\, R_s\, \varepsilon_{ks} \right]^{1/2}$$

For constant damping ratio

$$\varepsilon_{ks} = \left[\frac{(\omega_k + \omega_s)^2 \, \xi^2}{(\omega_k - \omega_s)^2 + (\omega_k + \omega_s)^2 \, \xi^2} \right]$$

For different damping ratio

$$\varepsilon_{ks} = \left[\frac{2\sqrt{\xi_k \xi_s} \, \left[(\omega_k + \omega_s)^2 \, (\xi_k + \xi_j) + (\omega_k - \omega_s)^2 \, (\xi_k - \xi_j) \right]}{4(\omega_k{}^2 - \omega_s{}^2) + (\omega_k + \omega_s)^2 \, (\xi_k + \xi_j)^2} \right]$$

A.2.3 Missing Mass Correction

In response spectrum analysis method we do the analysis for finite number of modes with cut-off frequency as 33Hz. In effect, the truncation of the mode series means that some mass of the system is ignored and is called as missing mass. Forces associated with these inertial masses are significant for system where mass participation at cut-off frequency is not enough. This missing mass is evaluated and forces are calculated by static method based on the following equations.

Mass Participated in the analysis $Mp = M \sum_{i=1}^{m} \Gamma_i \{\Phi_i\}$

Mass not Participated in the analysis (Missing Mass) $Ms = M \left\{ \{1\} - \sum_{i=1}^{m} \Gamma_i \{\Phi_i\} \right\}$

Force due to missing mass $= M\left\{\{1\} - \sum_{i=1}^{m}\Gamma_i\{\Phi_i\}\right\}S_{A\max}$

where, $S_{A\max}$ = highest spectral acceleration at the cut-off frequency
M = Total mass

A.2.4 Combination of Response due to Missing Mass with Dynamic Response

Residual response due to missing mass is combined with response due to dynamic analysis. The above response will be considered as an additional mode having frequency equal to the ZPA or cut-off frequency and will be combined using the SRSS rule.

A.2.5 Time history analysis

Let us consider the general case of a vibration problem

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\}$$

where, M is mass matrix (lumped/consistent), C is damping co-efficient matrix and K is stiffness matrix as also explained above.

Considering as a multi degree freedom system the equation of motion for the base excitation is written as

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}\}(t) + [K]\{x(t)\} = -[M]\ddot{x}_g(t)\{1\} \tag{A.4.1}$$

The above equation can be directly solved for responses using direct step by step integration techniques. For modal superposition technique, the following steps should be followed:

$$Let\{x(t)\} = \{\phi\}X(t)$$

Where $\{\phi\}$ is the mode shape and $X(t)$ is the generalized displacement

Substituting the above equation in previous equation,

$$we\ get\ [M]\{\phi\}\ddot{X}(t) + [C]\{\phi\}\dot{X}(t) + [K]\{\phi\}X(t) = -[M]\ddot{x}_g(t)\{1\}$$

Multiplying the above equation with $\{\phi\}^T$ on both sides we get

$$\{\phi\}^T[M]\{\phi\}\ddot{X}(t) + \{\phi\}^T[C]\{\phi\}\dot{X}(t) + \{\phi\}^T[K]\{\phi\}X(t) = -\{\phi\}^T[M]\ddot{x}_g(t)\{1\}$$

Using orthogonal properties of mode shapes, above equation can be written as

$$[1]\ddot{X}_n(t) + [2\varsigma\omega_n]\dot{X}_n(t) + [\omega_n^2]X_n(t) = -\Gamma_n\ddot{x}_g(t)$$

It can be seen that the above equations are un-coupled and solved separately for each modal properties of frequency and damping. For total response algebraic summation is performed. For design maximum values are considered.

To solve the equation of motion (equation (A.4.1)), step by step procedure can be adopted, in which the loading and the response history are divided into sequence time intervals or steps. The response during each step is then calculated from the initial conditions existing at the beginning of each time step and from the loading history during the step. These can be explicit or implicit.

Explicit methods are those in which new response quantities calculated in each step depend on quantities obtained in previous steps and analysis proceeds directly from one step to other.

In **Implicit methods** the expressions giving the new values for a given step include one or more values pertaining to that same step, so that trial values of necessary quantities must be assumed and then refined by iterations.

A.2.6 Time integration methods

This type of approach makes use of integration to step forward from the initial to the final conditions for each time step. The following equations represent the essential concept

$$\dot{x}_1 = \dot{x}_0 + \int_0^{\Delta t} \ddot{x}(\tau) d\tau$$

$$x_1 = x_0 + \int_0^{\Delta t} \dot{x}(\tau) d\tau$$

In order to carry out this type of analysis, it is necessary first to assume how the acceleration varies during the time step.

Some of the time integration techniques are Newmark beta technique and Wilson theta technique.

A.2.7 Response spectrum analysis

The un-coupled equations of a multi degrees of freedom system under base excitation are:

$$[1]\ddot{X}_n + [2\varsigma\omega_n]\dot{X}_n + [\omega_n^2]X_n = -\Gamma_n \ddot{x}_g$$

In the above equation, the matrices are diagonal and hence represents 'n' uncoupled equations of motion and each represents a single degree of freedom system with a factor on right hand side called participation factor. It means that if we know the response of single degree of freedom system and by multiplying it with participation factor, we can get the modal response of multi degree of freedom system as

$$\{\ddot{x}\}_n = \Gamma_n \{\phi\}_n \ddot{X}_n$$

where, $\ddot{X}_n$ are the spectral values as S_{a1}, S_{a2} etc corresponding to frequencies f_1, f_2 etc.

Hence, it can be concluded that once the frequencies are evaluated for multi degree of freedom system, the response accelerations can be obtained at that frequency. Once the generalized co-ordinates (product of spectral accelerations and participation factors) are obtained, then the responses at various nodes can be obtained multiplying it by mode shape.

A.2.8 Simplified methods

Two simplified methods can be used. The first one is the equivalent static method, when floor response spectrum is available and the second simplified method can be used in the absence of floor response spectrum as explained in. It is recommended to use these simplified methods for category-3 and category-4 equipment only.

A.2.8.1 Simplified method-1 (equivalent static method)

The equivalent static method is a simplified method as compared to any other dynamic analysis methods.

The equivalent static method may be applied to single point of attachment cantilever models with essentially uniform mass distribution.

The equivalent static load shall be determined by multiplying the structure, equipment or component masses by acceleration equal to 1.5 times the peak acceleration of the applicable response spectrum. Smaller value may be used if justified, or the floor ZPA value may be used if it is shown that

fundamental frequency is so high, typically 33 Hz. No dynamic amplification will occur at this frequency.

To obtain an equivalent static load for a structure, equipment, or component, which can be represented by a simple model, a factor of 1.5 is to be applied to the peak acceleration of the applicable floor response spectrum. A factor less than 1.5 can be used if adequate justification is provided.

The significance of utilizing factor 1.5 attributed to account for higher modes and add conservatism in the equivalent static method.

F_h = $K\ Sa_{max}\ W$

F_v = $K\ Sa_{max}\ W$

Where:

F_h = Equivalent static inertia force applied to the component in the horizontal direction

F_v = Equivalent static inertia force applied to the component in the vertical direction

K= load coefficient applied to the model. The value of K is taken as 1.5.

Sa_{max} = peak acceleration of applicable amplified or floor response spectra in the ith direction (in g's)

W = the total dead load (weight) which exist during the postulated seismic event. This includes piping weight, water weight and insulation. The units of the term W must be consistent with the terms F_h and Fv above.

2.8.2 Simplified method-2

The second simplified method can be used in the absence of Floor Response Spectrum (FRS) for the design of equipment; the following equation can be used to include the effect of floor response:

$$F_h = A_h \left(1 + 3\frac{h}{H}\right) W$$

where:

A_h is the design horizontal seismic coefficient (IS-1893: Part-1), which is given by

$$A_h = \frac{\left(\dfrac{Z}{2}\right)\left(\dfrac{S_a}{g}\right)}{\left(\dfrac{R}{I}\right)}$$

$\dfrac{S_a}{g}$ is design acceleration coefficient for different soil types, normalized with peak ground acceleration corresponding to natural period 'T' of the structure (in the absence of information on 'T', $\dfrac{S_a}{g} = 2.5$)

h is the height of the equipment location and H is the total height of the building

The factor $\left(1 + 3\dfrac{h}{H}\right)$ is a simplification to account for floor amplification.

W = the total dead load (weight) which exist during the postulated seismic event.

Example-1: A vessel of mass 5 Ton is mounted on the roof of a structure in seismic zone-II, as shown in Figure 4.1. Calculate the anchorage force if the vessel is a) supported on roof b) supported at a height half of the total structure height. Consider, $\dfrac{S_a}{g} = 2.5$, R=1 and I=1.
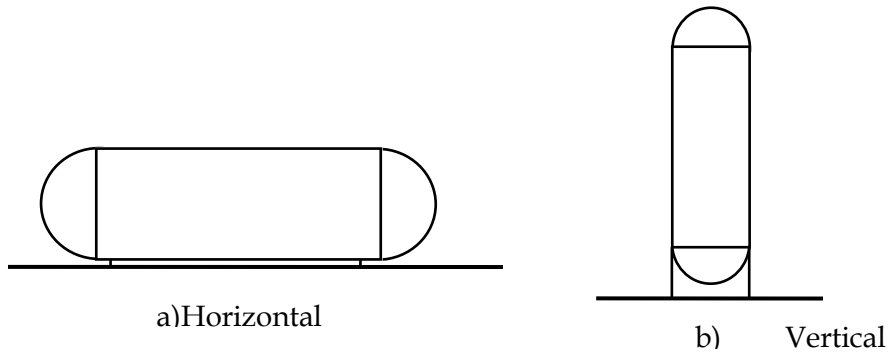
a)Horizontal   b)   Vertical

Figure 2.1 Schematic of a vessel mounted on the roof

For seismic zone-II, Z=0.1g;

Case-a: supported on roof

$$A_h = \frac{\left(\dfrac{Z}{2}\right)\left(\dfrac{S_a}{g}\right)}{\left(\dfrac{R}{I}\right)} = 0.05 x 2.5 = 0.125g$$

$$F_h = A_h\left(1 + 3\frac{h}{H}\right)W = (0.125 x 9.81) x (1+3)(5000 x 9.81) = 240.6kN$$

Case b: supported on ground

$$A_h = \frac{\left(\dfrac{Z}{2}\right)\left(\dfrac{S_a}{g}\right)}{\left(\dfrac{R}{I}\right)} = 0.05 x 2.5 = 0.125g \quad A_h = \frac{\left(\dfrac{Z}{2}\right)\left(\dfrac{S_a}{g}\right)}{\left(\dfrac{R}{I}\right)} = 0.05 x 2.5 = 0.125g$$

$$F_h = A_h\left(1 + 3\frac{h}{H}\right)W = (0.125 x 9.81) x (1+0)(5000 x 9.81) = 60.15kN$$

A.4.9 Procedure for calculating the hydrodynamic loads

The procedure for calculating the hydrodynamic load is given in IS-1893: Part-2 for Liquid Retaining Tanks.

Annexure- 3 Multi supported piping systems

The piping system shown in Fig.A.3.11 has supports at three locations such as one with top equipment and second at bottom equipment and laterally constrained at floor level. For seismic design of this piping system the basic inputs required are floor response spectra (FRS) at support locations and support displacements. If the supports are effective in three directions, then at each support three FRS are need to be considered and similarly three support displacements. Support displacements are also called Seismic Anchor Motions (SAM) need to be considered in design and corresponding stresses can be obtained by performing static analysis.
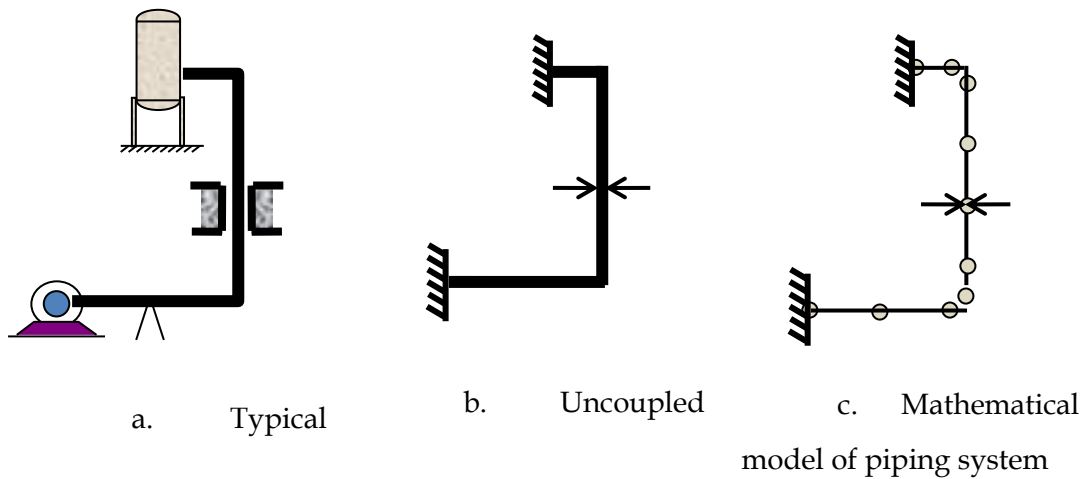

a.   Typical   b.   Uncoupled   c.   Mathematical
                                                model of piping system

Fig.A.3.1 Typical piping system connected to

The piping system may be decoupled from the equipment if the following criteria are satisfied.

a. Moment of inertia of equipment is more than 100 times the moment of inertia of the piping system
b. Equipment side ends of piping system have constraints along three directions. This requirement may be at one location or different locations.

After finalising the boundaries as shown in Fig.A.3.1 (b), suitable mathematical may be developed as shown in Fig.3.1(c) and analysed for the forces, moment and stresses. For selecting number of elements and type of elements such as straight pipe, bend , tee etc refer to the documents related to multi degree of freedom and piping design mentioned in the foreword. After finalising the model, following equilibrium equation is solved considering the piping subjected to multi support excitation.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = [M]\{\ddot{x}_g\}\{I_j\}$$ A.3.1

It is important to note that in Eq.A7.1 the influence vector $\{I_j\}$ has static displacements with unit displacement at the support j. It results in number of influence vectors equal to translational supports. Where as for uniform excitation the influence vector has 1s and 0s. Kindly note that the sum of all the influences at given node will

results in unity. The above equation cam be solved either for the time wise response or frequency (mode) wise response based on the input in terms of time history and response spectrum respectively. Usually later procedure is adopted since it is simple and Broadened FRS can be used without variations accounting for soil-structure-interaction, numerical solution variations and structure-equipment interactions. The modal response can be combined using suitable methods such as Square Root of Sum of Square, Absolute sum, 10% sum or Complete Quadratic Combination(CQC).

Annexure -4 Compatibility of Time Histories

One or more recorded, modified recorded or synthetic earthquake ground motions may be used for seismic response evaluation of structures using time history methods.

The time histories shall be selected to reasonably represent the Design Basis Spectra.

The time histories have to satisfy the requirements as follows.

i. The mean of the Zero period acceleration (ZPA) values calculated from the individual time histories shall equal or exceed the design basis ground displacement, velocity and acceleration

ii. In the frequency range of interest for design of structures, equipment and piping systems, the average ratios of spectrum to the design spectrum where the ratios are calculated frequency by frequency shall be equal to or greater than 1.

iii. No one point of the mean spectrum shall be more than 10% below the design spectrum.

iv. The average power spectral densities computed from the individual time histories shall be shown to possess adequate power at all frequencies in the frequency range of interest.

v. Response spectral values from any time histories shall be calculated at sufficient frequency points to produce accurate response spectra. Frequencies of structure, equipment and piping may be included in the list.

vi. When response from the three components of motion is calculated simultaneously on a time history basis, the input motion in the three directions shall be statistically independent. Two histories are considered statistically independent if the absolute value of the correlation coefficient does not exceed 0.3. For two time histories it is calculated as follows.

$$\rho_{1,2} = \frac{\sum (x_1 - m_1)(x_2 - m_2)}{\sqrt{\sum (x_1 - m_1)^2 \sum (x_2 - m_2)^2}}$$

Where E= Mathematical expectation

m_1 and m_2 are the mean values of x_1 and x_2